Tangent Lines

1999 (No Calculator)

6) Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

b) Write an equation of each horizontal tangent line to the curve.

c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y coordinates of point P.

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4) Suppose that the function f has a continuous second derivative for all x, an that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

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5) Consider the curve given by $xy^2 - x^3y = 6$.

a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.