AP Review: Separating of Variables / Slope Fields

1997 AB - 6, BC - 6 (Calculator)

6) Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

1998 (Calculator)

4) Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope if given by $\frac{3x^2 + 1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$.

b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

d) Use your solution from part (c) to find $f(1.2)$.

2000 (No Calculator)

6) Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

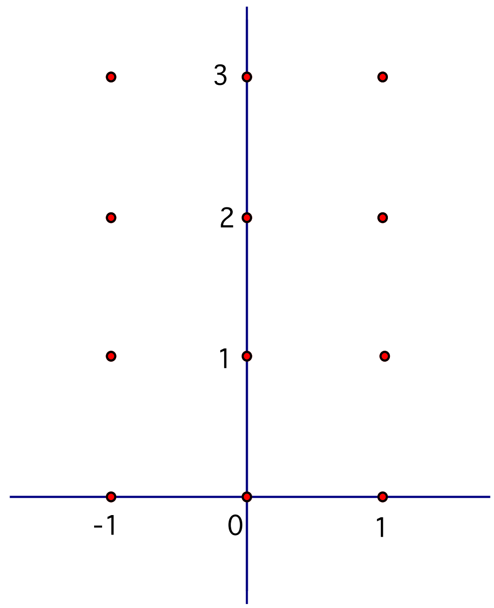
a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

b) Find the domain and range of the function f found in part (a).

2004 (No Calculator)

6) Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

a) On the axes provided sketch a slope field for the differential equation at the twelve points indicated.



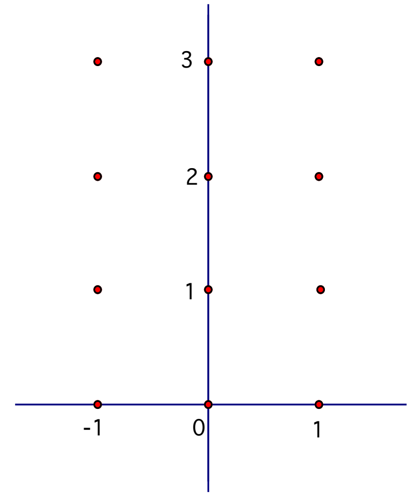
b) While the slope field in part a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

2004 (Form B) (No Calculator)

6) Consider the differential equation $\frac{dy}{dx} = x^4(y-1)$.

a) On the axes provided sketch a slope field for the differential equation at the twelve points indicated.



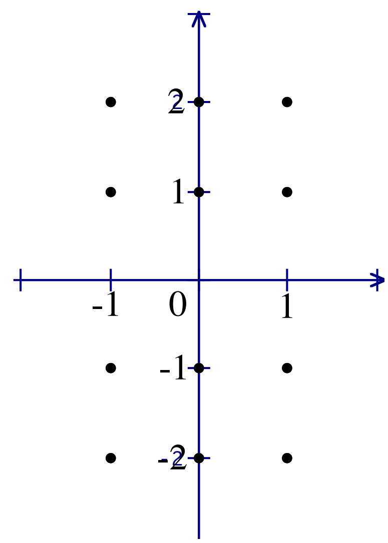
b) While the slope field in part a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are negative.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

2005 (No Calculator)

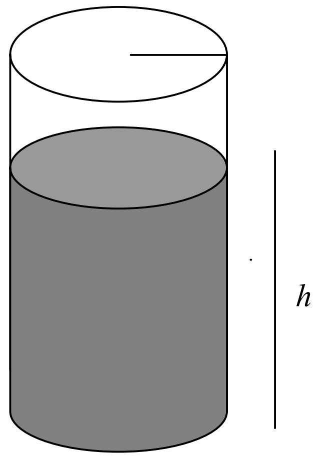
6) Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1,-1)$ and use it to approximate $f(1.1)$.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.



5) A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.

a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

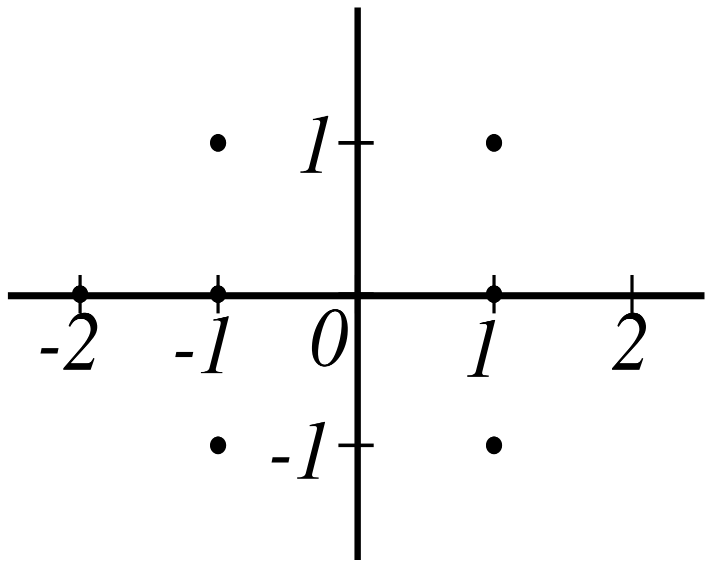
b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

c) At what time t is the coffeepot empty?

2006 (No Calculator)

5) Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

2010 **(No Calculator)**

Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.