CW ENTER / LEAVE PROBLEMS

2002 (Calculator)

2) The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t)=\frac{15600}{(t^2-24t+160)}$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t)=\frac{9890}{(t^2-38t+370)}$$

Both and are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t=9$, there are no people in the park.

a) How many people have entered the park by 5:00 P.M. ($t=17$)? Round your answer to the nearest whole number.

b) The price of admission is \$15 until 5:00 P.M. ($t=17$)? After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on a given day? Round your answer to the nearest whole number.

c) Let $H(t)=\int_9^t (E(x)-L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

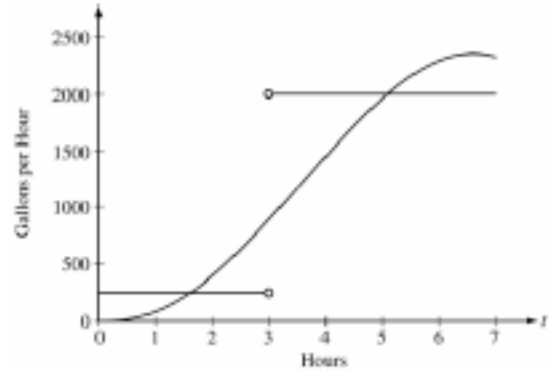
2007 #2 (Calculator)

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

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(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

2009 #2 (Calculator)

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t)=1380t^2-675t^3$ for $0\le t\le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t=0$, when the doors open. The doors close and the concert begins at time $t=2$.

a) How many people are in the auditorium when the concert begins?

b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t)=(2-t)R(t)$. Find $w(2)-w(1)$, the total wait time for those who enter the auditorium after time $t=1$.

d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part *c*.

2010 (Calculator)

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?