CH.2 Related Rates WS **Name :** ______________________________ **Per.** ____

1) Find $\frac{dx}{dt}$ given $x = 5$ and $\frac{dy}{dt} = 7$ for the equation $3x^2 - 5y^3 = 35$.

$3(5)^2 - 5y^3 = 35$ $\quad y = 2$ $\quad 6x\frac{dx}{dt} - 15y^2\frac{dy}{dt} = 0$ $\quad 6(5)\frac{dx}{dt} - 15(2)^2(7) = 0$ $\quad \underline{\frac{dx}{dt} = 14}$

2) The radius of a circle is increasing at the rate of 4 feet per minute.

a) Find the rate at which the area $(A = \pi r^2)$ is increasing when the radius is 12 feet. $\underline{\frac{dA}{dt} = 96\pi \ \text{ft}^2/\text{min.}}$

b) Find the rate at which the circumference $(C = 2\pi r)$ is increasing at the same time. $\underline{\frac{dC}{dt} = 8\pi \ \text{ft}/\text{min.}}$

a) $A = \pi r^2$ $\quad \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ $\quad \frac{dA}{dt} = 2\pi(12)(4)$ $\quad \frac{dA}{dt} = 96\pi \ \text{ft}^2/\text{min.}$

b) $C = 2\pi r$ $\quad \frac{dC}{dt} = 2\pi\frac{dr}{dt}$ $\quad \frac{dC}{dt} = 2\pi(4)$ $\quad \frac{dC}{dt} = 8\pi \ \text{ft}/\text{min.}$

3) A spherical balloon $\left(V = \frac{4}{3}\pi r^3\right)$ is inflated at the rate of 11 cubic feet per minute.

a) How fast is the radius of the balloon changing at the instant the radius is 5 feet? $\underline{\frac{dr}{dt} = \frac{11}{100\pi} \ \text{ft}/\text{min.}}$

b) How fast is the surface area $(A = 4\pi r^2)$ of the balloon changing at the same time? $\underline{\frac{dA}{dt} = \frac{22}{5} \ \text{ft}^2/\text{min.}}$

a) $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ $\quad 11 = 4\pi(5)^2\frac{dr}{dt}$ $\quad \frac{dr}{dt} = \frac{11}{100\pi} \ \text{ft}/\text{min.}$

b) $A = 4\pi r^2$ $\quad \frac{dA}{dt} = 8\pi r\frac{dr}{dt}$ $\quad \frac{dA}{dt} = 8\pi(5)\frac{11}{100\pi}$ $\quad \frac{dA}{dt} = \frac{22}{5} \ \text{ft}^2/\text{min.}$

4) The height of a cylinder with a radius of 4 ft. is increasing at a rate of 2 feet per minute.
Find the rate of change of the volume of the cylinder when the height is 6 feet. $(V = \pi r^2 h)$

$V = \pi r^2 h$ $\quad V = 16\pi h$ $\quad \frac{dV}{dt} = 16\pi\frac{dh}{dt}$ $\quad \frac{dV}{dt} = 16\pi(2)$ $\quad \underline{\frac{dV}{dt} = 32\pi \ \text{ft}^3/\text{min.}}$

5) A conical tank is 20 feet across the top and 15 feet deep. If water is flowing into the tank at the rate of 9 cubic feet per minute, $\left(V=\frac{1}{3}\pi r^2 h\right)$

a) find the rate of change of the depth of the water the instant that it is 2 feet deep. $\frac{dh}{dt}=\frac{81}{16\pi}\ \text{ft}/\text{min.}$

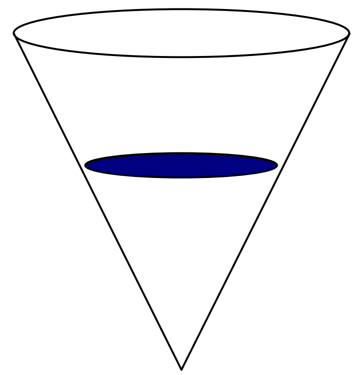
b) find the rate of change of the surface of the water at the same time. $\frac{dA}{dt}=9\ \text{ft}^2/\text{min.}$

a) $\frac{r}{h}=\frac{10}{15}$ $\quad r=\frac{2}{3}h$ $\quad V=\frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h$ $\quad V=\frac{4}{27}\pi h^3$

$\frac{dV}{dt}=\frac{4}{9}\pi h^2\frac{dh}{dt}$ $\quad 9=\frac{4}{9}\pi(2)^2\frac{dh}{dt}$ $\quad \frac{dh}{dt}=\frac{81}{16\pi}\ \text{ft}/\text{min.}$

b) $A=\pi r^2$ $\quad A=\pi\left(\frac{2}{3}h\right)^2$ $\quad A=\frac{4}{9}\pi h^2$

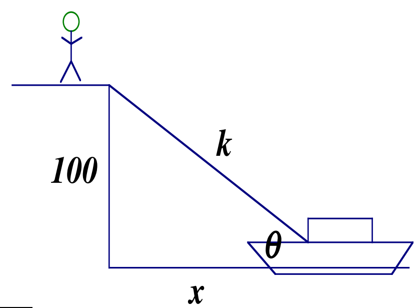
$\frac{dA}{dt}=\frac{8}{9}\pi h\frac{dh}{dt}$ $\quad \frac{dA}{dt}=\frac{8}{9}\pi(2)\frac{81}{16\pi}$ $\quad \frac{dA}{dt}=9\ \text{ft}^2/\text{min.}$



6) A man standing on a 100 ft. cliff watches a boat heading away from the cliff. The boat is travelling at a rate of 88 ft/s.

a) How fast is the distance k between the boat and the man changing when the boat is 70 ft. from the cliff? 50.465 ft/s.

b) How fast is the angle θ changing at this time? $-0.591\ \text{radians}/\text{s}$



$x^2+10000=k^2$ $\quad 2x\frac{dx}{dt}=2k\frac{dk}{dt}$ $\quad 2(70)(88)=2\left(\sqrt{14900}\right)\frac{dk}{dt}$

$\sin\theta=\frac{100}{k}$ $\quad \cos\theta\frac{d\theta}{dt}=\frac{-100}{k^2}\frac{dk}{dt}$ $\quad \frac{70}{\sqrt{14900}}\frac{d\theta}{dt}=\frac{-100}{14900}\left(\frac{6160}{\sqrt{14900}}\right)$

7) A plane is travelling toward an observer at 300 mph. The plane is flying 3 miles above the ground.

a) How fast is the distance m between the plane and the man changing when the plane is 5 miles from the man $(m=5)$? $\frac{dx}{dt}=-240$ mph

b) How fast is the angle of depression θ changing at this time? $\frac{d\theta}{dt}=36\ \text{radians}/\text{hr}$

$x^2+9=m^2$ $\quad 2x\frac{dx}{dt}=2m\frac{dm}{dt}$ $\quad 2(4)(-300)=2(5)\frac{dm}{dt}$

$\tan\theta=\frac{3}{x}$ $\quad \sec^2\theta\frac{d\theta}{dt}=\frac{-3}{x^2}\frac{dx}{dt}$ $\quad \left(\frac{5}{4}\right)^2\frac{d\theta}{dt}=\frac{-3}{4^2}(-300)$