$\qquad$

| $t$ <br> (minutes) | $R(t)$ <br> gallons/min. |
| :---: | :---: |
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |



2003
(Calculator)
2) The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice differentiable and strictly increasing function R of time $t$. The graph of $R$ and a table of selected values of $R(t)$ for the time interval $0 \leq t \leq 90$ minutes, are shown above
a) Use data from the table to find an approximation for $R^{\prime}(45)$. Show the computations that lead to your answer. Indicate units of measure.
b) The rate of fuel consumption is increasing fastest at time $t=45$ minutes. What is the value of $R^{\prime \prime}(45)$ ? Explain your reasoning.
c) Approximate the value of $\int_{0}^{90} R(t) d t$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_{0}^{90} R(t) d t$ ? Explain your reasoning.
d) For $0<b \leq 90$ minutes, explain the meaning of $\int_{0}^{b} R(t) d t$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_{0}^{b} R(t) d t$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

## 2004 (Calculator)

1) Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$
F(t)=82+4 \sin \left(\frac{t}{2}\right) \text { for } 0 \leq t \leq 30
$$

where $F(t)$ is measured in cars per minute and $t$ is measured in minutes.
a) To the nearest whole number, how many cars pass through the intersection over the 30 -minute period?
b) Is the traffic flow increasing or decreasing at $t=7$ ? Give a reason for your answer.
c) What is the average traffic flow over the time interval $10 \leq t \leq 15$ ? Indicate units of measure.
d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$ ? Indicate units of measure.

| Distance <br> $\mathrm{x}(\mathrm{cm})$ | 0 | 1 | 5 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature <br> $\mathrm{T}(\mathrm{x})(\mathrm{C})$ | 100 | 93 | 70 | 62 | 55 |

3) A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius $\left({ }^{\circ} \mathrm{C}\right)$, of the wire $x \mathrm{~cm}$ from the heated end. The function $T$ is decreasing and twice differentiable.
a) Estimate $T^{\prime}(7)$. Show the work that leads to your answer. Indicate units of measure.
b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
c) Find $\int_{0}^{8} T^{\prime}(x) d x$, and indicate units of measure. Explain the meaning of $\int_{0}^{8} T^{\prime}(x) d x$ in terms of the temperature of the wire.
d) Are the data in the table consistent with the assertion that $T^{\prime \prime}(x)>0$ for every $x$ in the interval $0<x<8$ ? Explain your answer.

## Question 2

| $t$ <br> (hours) | 0 | 2 | 5 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E(t)$ <br> (hundreds of <br> entries) | 0 | 4 | 13 | 21 | 23 |

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t=0)$ and 8 P.M. $(t=8)$. The number of entries in the box $t$ hours after noon is modeled by a differentiable function $E$ for $0 \leq t \leq 8$. Values of $E(t)$, in hundreds of entries, at various times $t$ are shown in the table above.
(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t=6$. Show the computations that lead to your answer.
(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8} \int_{0}^{8} E(t) d t$. Using correct units, explain the meaning of $\frac{1}{8} \int_{0}^{8} E(t) d t$ in terms of the number of entries.
(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function $P$, where $P(t)=t^{3}-30 t^{2}+298 t-976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight $(t=12)$ ?
(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

