# <u>Approximations / Charts CW</u> <u>(Trapezoidal Rule / Riemann Sums)</u>

**<u>1996</u>** (Calculator)

3) The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where $S$ is measured in billions of gallons per year and $t$ is measured in years from the beginning of 1980.

a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find $C$ and $k$.

b) Find the average rate of consumption of cola over the 10-year time period beginning January 1,1983. Indicate units of measure.

c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

d) Using correct units, explain the meaning of $\int_5^7 \mathrm{S}(\mathrm{t})\,dt$ in terms of cola consumption.

| $t$ (hours) | $R(t)$ (gallons per hour) |
|---|---|
| 0 | 9.6 |
| 3 | 10.4 |
| 6 | 10.8 |
| 9 | 11.2 |
| 12 | 11.4 |
| 15 | 11.3 |
| 18 | 10.7 |
| 21 | 10.2 |
| 24 | 9.6 |

**1999** (Calculator)

3) The rate at which water flows out a pipe, in gallons per hour, is given by a differentiable function $R$ of time $t$. The table above shows the rate as measured every 3 hours for a 24-hour period.

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

b) Is there some time $t$, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$.
Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
Indicate units of measure.

| $t$ $(days)$ | $W(t)$ °C |
|---|---|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

**<u>2001</u>** (Calculator)

2) The temperature in degrees Celsius, of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

b) Approximate the average temperature, in degrees Celsius of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for temperature of the water in the pond at time $t$, where $t$ is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using approximate units, explain the meaning of your answer in terms of water temperature.

d) Use the function defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$.

**2004** (Form B) (Calculator)

1) For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time $t$ days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

c) According to the model, how many mosquitoes will be on the island at time $t = 31$?
Round your answer to the nearest whole number.

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$?
Show the analysis that leads to your conclusion.

# 2007 #5 (No Calculator)

| $t$ (minutes) | 0 | 2 | 5 | 7 | 11 | 12 |
|---|---|---|---|---|---|---|
| $r'(t)$ (feet per minute) | 5.7 | 4.0 | 2.0 | 1.2 | 0.6 | 0.5 |

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function $r$ of time $t$, where $t$ is measured in minutes. For $0 < t < 12$, the graph of $r$ is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius $r$ is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\, dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\, dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\, dt$ ? Give a reason for your answer.